Supplementary Material

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(A)THE REALIZATION AND THE CONVENTIONAL CHARACTERIZATION OF THE POST-SELECTION

We construct the symmetric informationally complete (SIC) positive-operator-valued measure (POVM) in the polarization degree of freedom (DOF) as the general quantum measurement to be characterized. In the direct tomography (DT) protocol, SIC POVM performs the post-selection of the quantum system. The detailed configurations of the SIC POVM are given in the Fig. 1. As a comparison to the results of DT, we first characterize the realistic SIC POVM using conventional quantum detector tomography (QDT). The input states are prepared by the horizontally polarizing photons passing through a half-wave plate and a quarter-wave plate. The generation of five input states $|\kappa_m\rangle$ and the obtained probabilities $P_l^{(m)}$ of the measurement outcome l are given in the Table. The POVM of the quantum measurement $\{\hat{\Pi}_l\}(l=1,2,3,4)$ can be reconstructed by optimizing

$$\hat{\Pi}_n = \arg\min \sum_{l,k} [P_l^{(k)} - \langle\kappa_m|\hat{\Pi}_l|\kappa_m\rangle]^2 \tag{1}$$

with the constraints $\hat{\Pi}_n \geq 0$ and $\sum_l \hat{\Pi}_l \geq 0$. The reconstructed POVM elements in the $\{|H\rangle, |V\rangle\}$ representation are given by

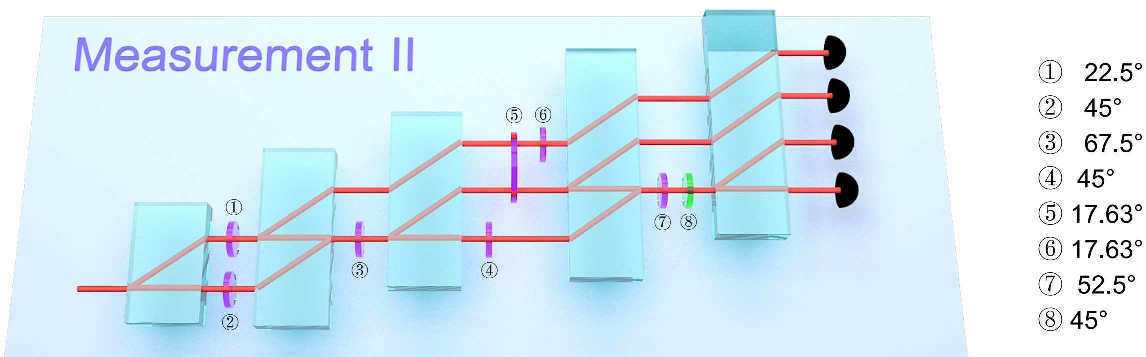

FIG. 1. The detailed configuration of module 'Measurement II'.

TABLE I. quantum detector tomography of the 'Measurement II'

	HWP	QWP	$P_1^{(i)}$	$P_2^{(i)}$	$P_3^{(i)}$	$P_4^{(i)}$
$\|\kappa_1\rangle$	45°	-	0.0005	0.3682	0.3513	0.2800
$\|\kappa_2\rangle$	17.63°	-	0.3681	0.0008	0.3193	0.3118
$\|\kappa_3\rangle$	−27.37°	−27.37°	0.3444	0.3443	0.3096	0.0017
$\|\kappa_4\rangle$	-	−27.37°	0.3439	0.3812	0.0018	0.2730
$\|\kappa_5\rangle$	-	-	0.5202	0.1892	0.1396	0.1510

To make the measured probabilities best described, we determine the POVM by the optimization

$$\hat{\Pi}_n = \arg\min \sum_{i,n} \left[P_n^{(i)} - \langle\kappa_i|\hat{\Pi}_n|\kappa_i\rangle\right]^2 \tag{2}$$

with the constraints $\hat{\Pi}_n \geq 0$ and $\sum_n \hat{\Pi}_n = \hat{I}$. The reconstructed POVM elements in the $\{|H\rangle, |V\rangle\}$ representation are given as follows

$$\hat{\Pi}_1 = \begin{pmatrix} 0.5241 & 0.0170 + 0.0002i \\ 0.0170 - 0.0002i & 0.0024 \end{pmatrix}, \tag{3}$$

$$\hat{\Pi}_2 = \begin{pmatrix} 0.1841 & -0.2559 - 0.0227i \\ -0.2559 + 0.0227i & 0.3656 \end{pmatrix}, \tag{4}$$

$$\hat{\Pi}_3 = \begin{pmatrix} 0.1396 & 0.1157 + 0.1884i \\ 0.1157 - 0.1884i & 0.3513 \end{pmatrix}, \tag{5}$$

$$\hat{\Pi}_4 = \begin{pmatrix} 0.1522 & 0.1232 - 0.1659i \\ 0.1232 - 0.1659i & 0.2806 \end{pmatrix}. \tag{6}$$